Title: Three Ways to Solve Two-Step Linear Equations.

Brief Overview:

This lesson provides three alternative methods for solving one-variable/two step equations. Students will move from concrete examples to abstract problem solving by using multiple modalities. Students will use mental math, physical and symbolic representations, and verbalization of skills.

NCTM Content Standard/National Science Education Standard:

Numbers and Operations – understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Algebra – represent and analyze mathematical situations and structures using algebraic symbols.

Problem Solving - apply and adapt a variety of appropriate strategies to solve problems.

Communication – communicate their mathematical thinking coherently and clearly to peers, teachers, and others. Use the language of mathematics to express mathematical ideas precisely.

Grade/Level:

Advanced 6^{th}/Pre-Algebra 7^{th}/Pre-Algebra 8^{th}

Duration/Length:

One 86-minute block per lesson.

Student Outcomes:

Students will:

- Simplify expressions by combining like terms and applying the order of operations.
- Solve linear equations in one variable using mathematical properties.
- Describe a real-world situation represented by an algebraic expression or equation.

Materials and Resources:

- Resource worksheets
- Algebra tiles – class sets and overhead
- Calculators

Development/Procedures:

Lesson 1

Preassessment – Begin the lesson with everyday situations in which the students are intuitively solving for an unknown.
"I have $60 dollars. How many CDs could I buy if they cost $15 each?"
Once the correct answer has been solicited from the class, the teacher should demonstrate this problem on the overhead/ board:

$$\frac{60}{15} = 4 \text{ CD's}$$

"How many eggs do I have if I have 4½ cartons full?"
4.5 * 12 = 54 eggs
"How many seconds are there in 2 hours?"
120 * 60 = 7200 seconds
"My recipe calls for 3 cups of flour, but I only want to make half. How much flour do I need?"
3 ÷ 2 = 1½ cups

Launch – Revisit the same expressions in the abstract, now that the students have demonstrated their competence with these basic operations, and help the students format their concepts in an algebraic form.
"I have *x* dollars. How many CDs could I buy if they cost $15 each?"

$$\frac{x}{15} =.$$

"How many eggs do I have if I've got *x* cartons full?"
$12x$
"How many seconds are there in *x* hours?"
$(x * 60) * 60 = 3600x$
"My recipe calls for *x* cups of flour, but I only want to make half. How much flour do I need?"

$$\frac{x}{2}$$

Teacher Facilitation – Help the students build the mental bridge between the concrete and the abstract by asking them which mathematical operations (addition, subtraction, multiplication and division) correspond with words like "more than" "twice as much" "decrease" etc. Hand out The Language of Algebra Cards. (Resource Master 1). Instruct them to find their "match". For example, a student with the "Half as much as a number plus three" will have to find the student with the card reading $\frac{x+3}{2}$. This is an

effective way of pairing up the students for the rest of the lesson.

Student Application – Have the students will now in turn stand and read out the "word" form of their equation and give their fellow students a few moments to write down their best guess as to how the formula version of the equation will look. Rather than "think pair share", use "think pair write" with students collaborating if necessary but then writing their work in their journal. Once the allotted time has passed, the student with the answer will share the correct form with the class on the overhead or on the board.

Embedded Assessment –Circulate through the class and monitor students' progress and understanding of the concepts. Hand out worksheets (Resource master 2). Students are encouraged to ask their partner for assistance but each student should complete his/her own worksheet.

Reteaching/Extension –

- For those who have not completely understood the lesson, review what is needed by using the reteaching worksheet, Resource Master 3.
- For those who have understood the lesson, teacher can have a 'challenge' problem, for example $5x - 12 - 2x = 3$

Lesson 2

Preassessment – Note that the students will be familiar with the language of algebraic expressions from the previous lesson. They should also have a working knowledge of positive and negative integers. They should be familiar with Algebra Tiles at least for use with positive and negative integers.

Launch – Review adding positive and negatives as well as questions based on solving one-step equations by using Resource Master 4.

Teacher Facilitation –Have the students tape or tie on the "Human Algebra Tiles," Resource 6. Problems are provided on Resource Master 5. Demonstrate that while a positive and negative "1" cancel each other, so do positive and negative "*x*'s ". Now a simple problem can be modeled for the class, such as "$2x + 3 = 7$". Arrange two positive "*x*" students and three positive "1" students on one side of the room, and seven more positive "1" students on the other side. The necessary "negative" students can be chosen from the rest of the class to pair off into zeros, understanding that the same number of negative ones must be added to the side with seven as the side with three. The "Human Equation" will now be $2x=4$. It can then be explained that students who are "*x*'s" must evenly divide the leftover "1's".

Since everything in the "Human Equation" is "fair", students should understand to cancel ones first, then evenly divide the remaining ones among the *x*'s.

Student Application – Using the resource master, have the students act out more example problems. They may then sit in groups and work the more difficult problems using Algebra Tiles. (If students are unfamiliar with Algebra Tiles, they may need some instruction).

Embedded Assessment – The teacher can observe students in groups using Algebra Tiles to solve problems. This should provide information as to whether there is need for more instruction or if students are ready to solve more difficult problems.

Reteaching/Extension –

- Using overhead algebra tiles, the teacher can work out some more practice problems with those who need re-teaching.
- The next step is to solve problems that involve numbers too large to represent with the tiles.

Lesson 3

Preassessment – Students have solved two-step equations with one variable. Students should be familiar with the order of operations. (If needed, order of operations problems could be reviewed in the warm up activity).

Launch – Have the students should recall solving problems with an unknown using Algebra Tiles. Today, through an opening activity, announce that they will find the unknown quantity by working backwards from the answer.

Teacher Facilitation – Present an overhead of "The Story of X" using Resource Master 7. Work through a few examples where students can discuss the missing operations and numbers. Use Resource Master 8 to facilitate the discussion.

Student Application – Ask the students to create a letter to a friend describing what "happened" to x. They should be creative, and the story in the letter should include mathematical operations and numbers. Stories can include the same operation more than once, but should include at least two steps. Students should be able to write the equation that goes with their story. This activity can be modified to use multiple steps with parentheses if students are prepared for more difficult problems. Use Resource Masters 9 and 10.

Embedded Assessment – The teacher can observe the students' understanding of algebraic language and operations as they write the story.

Reteaching/Extension –

- Simple examples showing addition and subtraction (and multiplication/division) "canceling" each other might be used to better illustrate what is being done with the unknown in the chart.
- Students showing mastery should be asked to design a story that uses division or exponents in the original equation.

Summative Assessment:

Students have written a letter to a friend describing the story of an unknown quantity. They should trade letters with someone else, and each student should reply to a letter. In the reply, students should demonstrate their knowledge of "undoing" operations. They should also show a symbolic way to solve the equation. The finished "return" letter demonstrates a student's ability to work backwards and to solve problems using multiple methods. Use Resource Master 11.

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Four times a number increased by six.

$4x+6$

Seven more than some number

$n + 7$

Two less than a number.

$k - 2$

Twelve times some number.

$12z$

Twice a number.

$2d$

Seventy-five percent of some number.

$.75y$

Thirty nine percent of a number.

$.39h$

Nine less than twice a number.

$2c - 9$

Four more than half a number.

$$\frac{g}{2}+4$$

Eight more than ten percent of a number.

$$8+.1d$$

Five less than three percent of a number.

$.03z - 5$

One less than the difference of two numbers.

$(c-h)-1$

Two more than the product of two numbers.

$ab + 2$

Twenty-nine miles for g gallons of gasoline.

$29g$

One-hundred fifty miles in h hours.

$$\frac{150}{h}$$

The product of a number and negative seven.

$$-7s$$

One third of a number plus three.

$$\frac{1}{3}h+3$$

The sum of twelve and some number times fourteen.

$$14z + 12$$

(Resource Master 2)

What does that *x* stand for, anyway?

Remember that each equation is telling you something about the relationship of the two sides. $7 = x - 3$

$7 = x - 3$ says "seven is the same as some number take away three". If the number is the same as seven once three has been subtracted, then it must be the same as three more than seven. Three more than 7 is 10. Could x be 10?

Replace *x* with ten and see if it works. $7 = 10 - 3$ does work, so ten is the right answer.

For each problem below, find the missing number, "x".

1. $12x = 36$

$x =$ _____

2. $15 = x - 2$

$x =$ _____

3. $20 = x + 4$

$x =$ _____

4. $500 = x/2$

$x =$ _____

5. $15 = 3x$

$x =$ _____

6. $-4x = 36$

$x =$ _____

7. $0 = -2x$

$x =$ _____

8. $0 = -2 + x$

$x =$ _____

9. $42 = -7x$

$x =$ _____

10. $32 = x - 16$

$x =$ _____

11. $-9 = 27/x$

$x =$ _____

12. $15x = 60$

$x =$ _____

13. $18 = 3 + x$

$x =$ _____

14. $x + 9 = 81$

$x =$ _____

15. $250 = x/4$

$x =$ _____

(Resource Master 3 ~ Reteaching)

Name _____ Period _____

Who are you, *x*?

There are only four kinds of one-step questions – Addition, Subtraction, Multiplication and Division.
15 = *x* + 2 is saying think of a number. Add two to that number and you get fifteen. What is that number? You have to find a number that is two less than fifteen – guess 13 and you're right.

1. 15 = *x* + 5 **2**. 0 = -8 + *x* **3**. 7 = 5 + *x*

x= _____ *x*= _____ *x*= _____

4. *x* + 8 = 56 **5**. *x* + 2 = 10 **6**. 60 = 45 + *x*

x= _____ *x*= _____ *x*= _____

Subtraction problems are the opposite of the addition problems. If you can do one, you can do the other. 15 = 20 – *x* is saying: "Twenty take away some number gets you fifteen". What number is the difference between fifteen and twenty? Once you've figured out that the numbers are five apart, replace *x* with five and see if the answer works.

7. 5 = *x* - 5 **8**. 10 = 27 - *x* **9**. 23 = 50 - *x*

x= _____ *x*= _____ *x*= _____

10. *x* - 8 = 12 **11**. *x* - 2 = 10 **12**. 30 = 45 - *x*

x= _____ *x*= _____ *x*= _____

Multiplication problems look hard, but they're not. $12x = 36$ is saying some number multiplied by twelve = 36. There are two ways of solving this. One is to start doing your twelve tables in your head. Twelve times one is twelve. Twelve times two is twenty-four. Twelve times three is thirty-six. Oh, so the x must be three. The other way to solve it is to look at the thirty-six and ask yourself how many times twelve goes in there. Divide 36 by 12 and you'll get three.

13. $4x = 48$ $x =$ _____

14. $4x = 12$ $x =$ _____

15. $42 = 6x$ $x =$ _____

16. $500 = 10x$ $x =$ _____

17. $36 = 3x$ $x =$ _____

18. $25x = 100$ $x =$ _____

If you can do multiplication, there is no division problem that can stump you. $100/x = 25$ is just a fancy way of saying "One Hundred divided by some number is 25." How many twenty-fives are there in a hundred? Think money. Four quarters to a dollar. Four twenty-fives in 100. So, 100/4 = 25.

19. $15/x = 3$ $x =$ _____

20. $32/x = 2$ $x =$ _____

21. $15 = 60/x$ $x =$ _____

22. $63/x = 9$ $x =$ _____

23. $49/x = 7$ $x =$ _____

24. $54/x = 9$ $x =$ _____

(Resource Master 4 ~ Opening Activity, Lesson 2)

Opening Actvity

Solve the following problems for the unknown quantity using mental math.

1. $6x = 12$

2. $3x = 15$

3. $2x = 26$

4. $x + 2 = 7$

5. $x - 4 = 10$

6. $x - 2 = 7$

Describe how you used mental math to find your answer. Use complete sentences.

(Resource Master 5 ~ Examples for use with Algebra Tiles)

Solve the following for "*x*" using students as a "Human Equation":

1. $2x + 5 = 7$
2. $2x + 1 = 7$
3. $4x - 2 = 6$
4. $4x - 3 = 9$
5. $5x - 1 = 14$
6. $3x - 4 = -1$

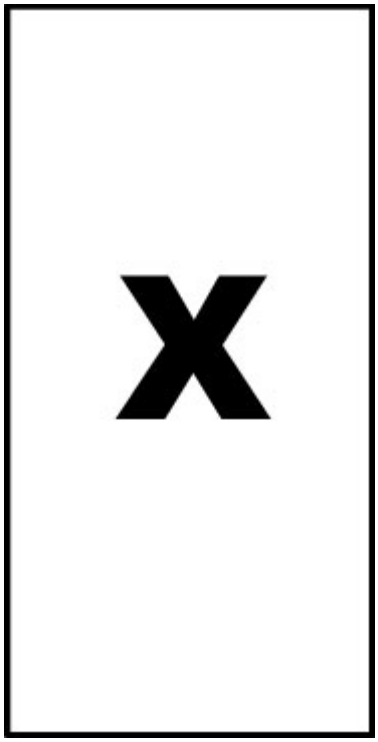
Solve the following for "*x*" using Algebra Tiles, (You may have to combine tiles with your group):

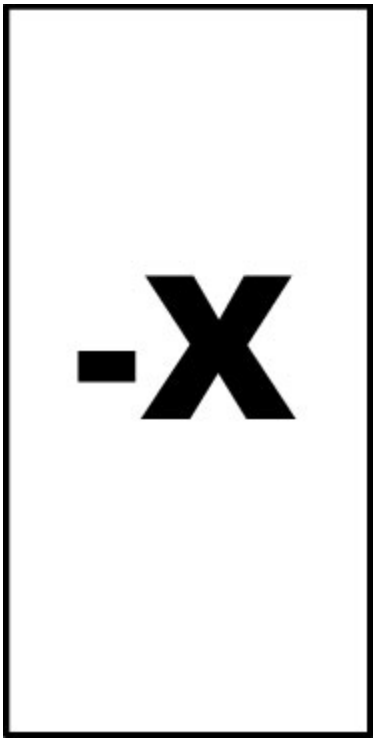
1. $6x + 7 = 13$
2. $4x - 2 = 6$
3. $6 + 2x = 12$
4. $12 + 3x = 6$
5. $3x - 4 = (-1)$
6. $2x + 4 + (-2) = 10$

Solve the following using any method you wish. Work on your own first, and then compare your answers with the group:

1. $3x + 6 = 24$
2. $7x - 3 = 18$
3. $8x + 7 = 31$
4. $4x - 12 = (-4)$
5. $2x - x + 4 = 8$
6. $2x - 2 = (-1)$
7. $\frac{1}{2}x + 4 = 8$ (think…)
8. $14x + 4 = 32$
9. $9x - 0 = 45$

(Resource Master 6 ~ Human Algebra Tiles Cards (enlarge and copy as needed))





(Resource Master 7 ~ The Story of x)

The Story of *x*

The Problem: 3*x* + 4 = 10

Let's make a list of what happened to '*x*' *before* it became 10.

	x
First, *x* was multiplied by 3.	***3**
Then, *x* had 4 added.	**+4**
The result to all of this was 10.	10

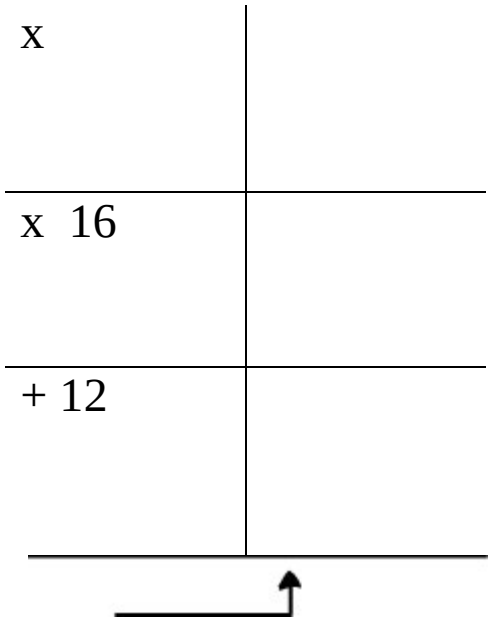
Now let's work backwards to find out what '*x*' was…

x	**2**	***x* used to be '2' !**
***3**	**/3**	**Divide by 3**
+4	**-4**	**Subtract 4**
10 ↑		**Follow the arrow and do the *opposite* of each operation.**

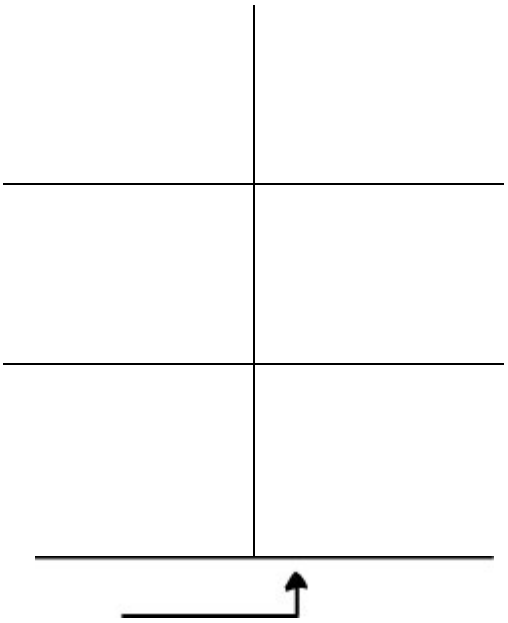
(Resource Master 8 ~ Practice Problems)

Try it! – Tell the story of x with the template below.

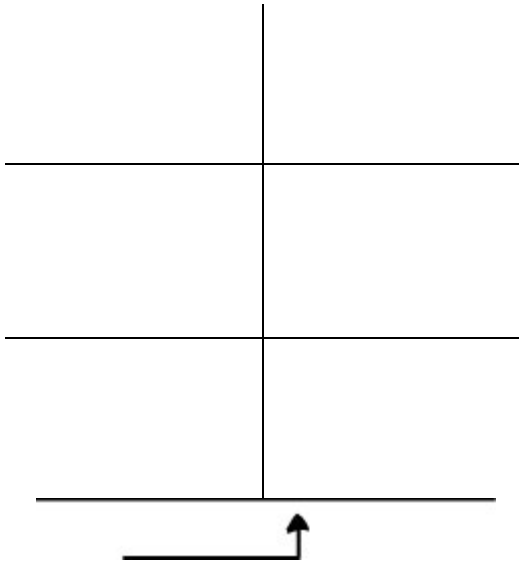
1. $16x + 12 = 44$



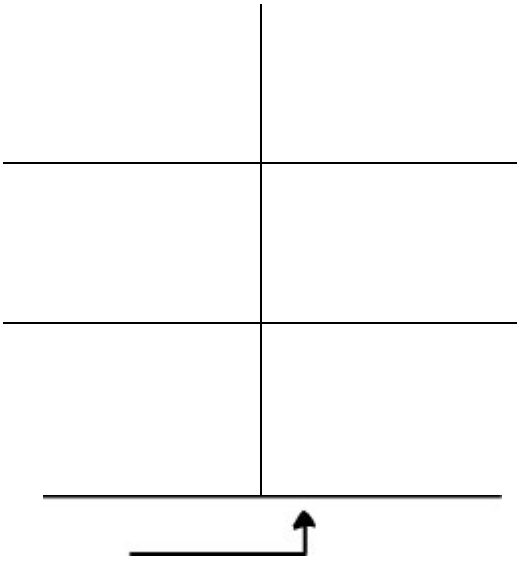
2. $22x - 6 = 5$



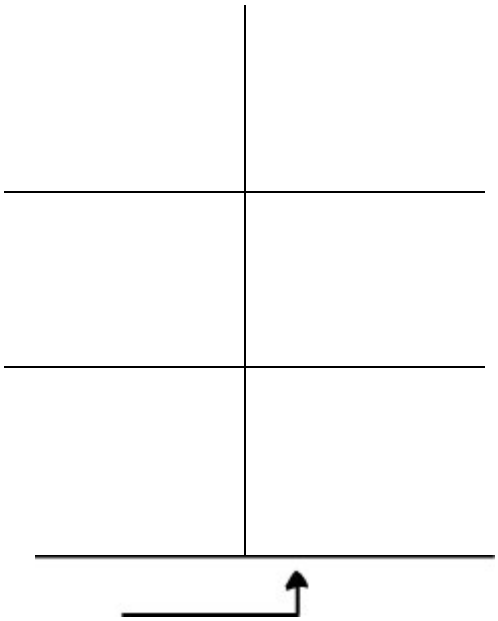
3. $\frac{1}{2}x + 9 = 23$

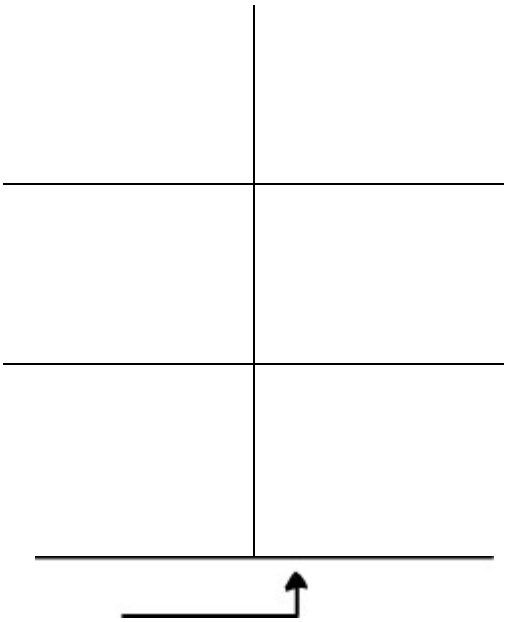


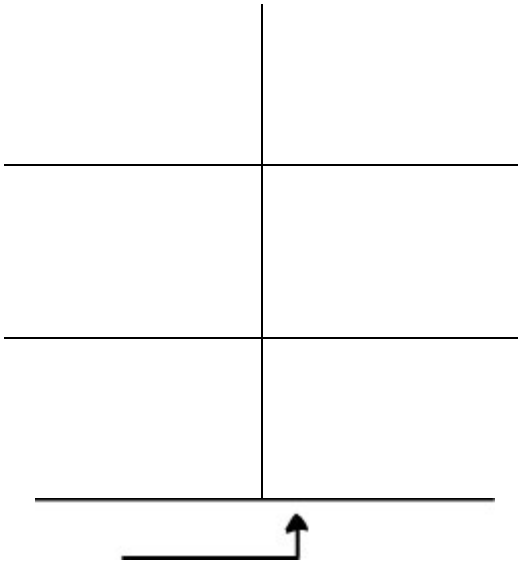
4. $7x - 9 = 40$

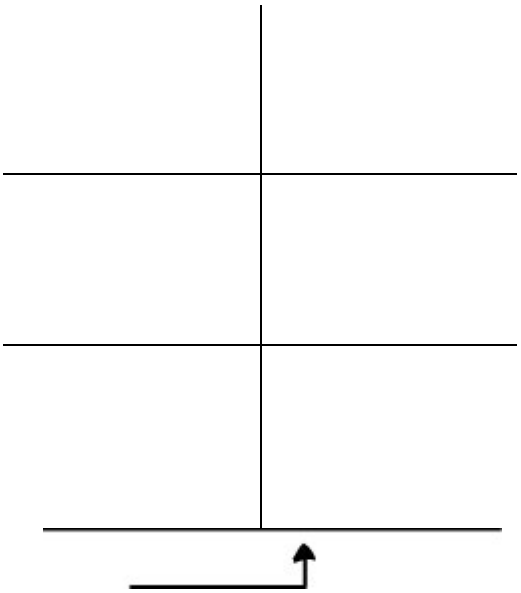


(Resource Master 9 ~ Blank Charts for the Story of 'x')









(Resource Master 10 ~ Write a Letter)

Write a letter to a friend

It's time to show off all your new skills! You must write a letter to a friend describing the story of '*x*'. You may be as creative as you like, but your letter must include the following items:

- At least two different types of operation (that's addition, multiplication etc.)
- At least three steps.
- Numbers that go with each operation.
- An equation that represents the story.
- You must write in complete sentences.

(Resource Master 11 ~ Reply to the Letter)

Reply

What was '*x*'? You must now reply to your friend. In your return letter you have to figure out what '*x*' was and explain how you found your answer. Your reply should be creative, and you must include the following items:

- Retrace the steps by undoing the operations.
- Explain each step.
- At least one *other* method for finding the unknown must be used. You may use charts, diagrams, Algebra Tiles, or mental math.
- All of your steps must be explained in complete sentences.

Answer Key

Resource 1 ~ In-Class Worksheet

1. **3**
2. **17**
3. **16**
4. **1000**
5. **5**
6. **–9**
7. **0**
8. **2**
9. **–6**
10. **48**
11. **–3**
12. **4**
13. **15**
14. **72**
15. **1000**

Resource 2 ~ Reteaching

1. **10**
2. **8**
3. **2**
4. **48**
5. **8**
6. **15**
7. **10**
8. **17**
9. **27**
10. **20**
11. **12**
12. **15**
13. **12**
14. **3**
15. **7**
16. **50**
17. **12**
18. **4**
19. **5**
20. **16**
21. **4**
22. **7**
23. **7**
24. **6**

Resource 4 ~ Opening Activity

1. **2**
2. **5**
3. **13**
4. **5**
5. **14**
6. **9**

Resource 5 ~ Examples

1. **1**
2. **3**
3. **2**
4. **3**
5. **3**
6. **1**

1. **1**
2. **2**
3. **3**
4. **–2**
5. **1**
6. **4**

1. **6**
2. **3**
3. **3**
4. **2**
5. **4**
6. **½**
7. **8**
8. **2**
9. **5**

Resource 6 ~ Practice Problems

1. **2**
2. **½**
3. **28**
4. **7**